Surds

1. Express $\frac{\sqrt{35}+3\sqrt{5}-2\sqrt{7}-6}{\sqrt{5}-2}$

in the form $m+\sqrt{n}$, where m and n are integers.
Show your working clearly.

$$\frac{\sqrt{35}+3\sqrt{5}-2\sqrt{7}-6}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{\sqrt{175}+2\cancel{\sqrt{35}}+15+6\cancel{\sqrt{5}}-2\cancel{\sqrt{35}}-4\sqrt{7}-6\cancel{\sqrt{5}}-12}{5-4}$$

$$= \frac{5\sqrt{7}+3-4\sqrt{7}}{1}$$

$$= 3+\sqrt{7}$$

$m = 3 \quad n = 1$

2. Without using a calculator and showing your working clearly, find the value of the integer a so that

$$\sqrt{180} - \sqrt{27} - \sqrt{20} + \sqrt{147} = a\,(\sqrt{5} + \sqrt{3}\,)$$

$\sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}$

$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$

$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$

$\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$

$6\sqrt{5} - 3\sqrt{3} - 2\sqrt{5} + 7\sqrt{3} = a(\sqrt{5} + \sqrt{3})$

$= 4\sqrt{5} + 4\sqrt{3} = a(\sqrt{5} + \sqrt{3})$

$4(\sqrt{5} + \sqrt{3}) = a(\sqrt{5} + \sqrt{3})$

$a = 4$

3. Without using a calculator and showing all your working, express $\frac{6-\sqrt{8}}{2+\sqrt{8}}$ in the form $a + \sqrt{b}$ where a and b are integers.

$$\frac{6-\sqrt{8}}{2+\sqrt{8}} \times \frac{2-\sqrt{8}}{2-\sqrt{8}}$$

$$= \frac{12 - 6\sqrt{8} - 2\sqrt{8} + 8}{4-8}$$

$$= \frac{20 - 8\sqrt{8}}{-4}$$

$$= \frac{8\sqrt{8} - 20}{4}$$

$$= \frac{\cancel{16}^{4}\sqrt{2} - \cancel{20}^{5}}{\cancel{4}}$$

$$= 4\sqrt{2} - 5$$

$$= \sqrt{16 \times 2} - 5$$

$$= \sqrt{32} - 5$$

$$= -5 + \sqrt{32}$$

4. Without using your calculator and showing all your working, express

$$\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$$

In the form $a + \sqrt{b}$ where a and b are integers.

$$\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}} \times \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}+\sqrt{11}}$$

$$= \frac{13+\sqrt{143}+\sqrt{143}+11}{13-11}$$

$$= \frac{24+2\sqrt{143}}{2}$$

$$= 12+\sqrt{143}$$

5. $\frac{2-\sqrt{2}}{(1+\sqrt{2})^2}$ can be written in the form $a + b\sqrt{2}$ where a and b are integers.

Find the value of a and the value of b.

Show your working clearly.

$$\frac{2-\sqrt{2}}{(1+\sqrt{2})^2} = \frac{2-\sqrt{2}}{(1+2\sqrt{2}+2)}$$

$$= \frac{2-\sqrt{2}}{(3+2\sqrt{2})}$$

$$\frac{2-\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{6-4\sqrt{2}-3\sqrt{2}+4}{9-8}$$

$$= \frac{10-7\sqrt{2}}{1}$$

$$= 10-7\sqrt{2}$$

$a = 10$ $\quad b = -7$

6. Without using a calculator and showing all your working, express

$$\frac{4-\sqrt{12}}{4+\sqrt{12}}$$

in the form $a - \sqrt{b}$ where a and b are integers.

$$\frac{4-\sqrt{12}}{4+\sqrt{12}} \times \frac{4-\sqrt{12}}{4-\sqrt{12}}$$

$$= \frac{16-4\sqrt{12}-4\sqrt{12}+12}{16-12}$$

$$= \frac{28-8\sqrt{12}}{4}$$

$$= \frac{28-16\sqrt{3}}{4}$$

$$= 7-4\sqrt{3}$$

$$= 7-\sqrt{48}$$

7. Without using a calculator, and showing all your working, express

$$\frac{\sqrt{75}+\sqrt{243}}{\sqrt{7}}$$

in the form $\sqrt{a}$ where a is a positive integer.
Show your working clearly.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$\sqrt{243} = \sqrt{81 \times 3} = 9\sqrt{3}$$

$$\frac{5\sqrt{3} + 9\sqrt{3}}{\sqrt{7}} \times \frac{-\sqrt{7}}{-\sqrt{7}}$$

$$= \frac{-5\sqrt{21} - 9\sqrt{21}}{7}$$

$$= \frac{-14\sqrt{21}}{7}$$

$$= -2\sqrt{21}$$

$$= \sqrt{21 \times 4}$$

$$= \sqrt{84}$$

8. Given that m and n are integers, write $\dfrac{4+\sqrt{20}}{\sqrt{5}-2}$ in the form $m + n\sqrt{5}$

Show each stage of your working clearly.

$$\frac{4+\sqrt{20}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{4\sqrt{5} + 8 + \sqrt{100} + 2\sqrt{20}}{5-4}$$

$$= \frac{4\sqrt{5} + 8 + 10 + 4\sqrt{5}}{1}$$

$$= 18 + 8\sqrt{5}$$

9. Given that $p = \frac{1+\sqrt{5}}{2}$

Show that $\frac{1}{p} = p - 1$

Show your working clearly.

$$\frac{1}{p} = \frac{2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{2-2\sqrt{5}}{1-5} = \frac{2-2\sqrt{5}}{-4} = \frac{1-\sqrt{5}}{-2} = \frac{\sqrt{5}-1}{2}$$

$$p-1 = \frac{1+\sqrt{5}}{2} - 1$$

$$= \frac{1+\sqrt{5}-2}{2}$$

$$= \frac{\sqrt{5}-1}{2}$$

$$\therefore \frac{1}{p} = p - 1$$

10. Without using a calculator and showing all your working, express

$$\frac{4-2\sqrt{3}}{\sqrt{3}+1}$$

in the form $a\sqrt{3}+b$ where a and b are integers.

$$\frac{4-2\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{4\sqrt{3}-4-6+2\sqrt{3}}{3-1}$$

$$= \frac{\overset{3}{\cancel{6}}\sqrt{3}-\overset{5}{\cancel{10}}}{\cancel{2}}$$

$$= 3\sqrt{3}-5$$