



Surds

1. Express $\frac{\sqrt{35} + 3\sqrt{5} - 2\sqrt{7} - 6}{\sqrt{5} - 2}$

in the form $m + \sqrt{n}$, where m and n are integers.

Show your working clearly.

$$\frac{\sqrt{35} + 3\sqrt{5} - 2\sqrt{7} - 6}{\sqrt{5} - 2} \times \frac{\sqrt{5} + 2}{\sqrt{5} + 2}$$

$$= \frac{\sqrt{175} + 2\sqrt{35} + 15 + 6\sqrt{5} - 2\sqrt{35} - 4\sqrt{7} - 6\sqrt{5} - 12}{5 - 4}$$

$$= \frac{5\sqrt{7} + 3 - 4\sqrt{7}}{1}$$

$$= 3 + \sqrt{7}$$

$$m = 3 \quad n = 7$$

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2. Without using a calculator and showing your working clearly, find the value of the integer a so that

$$\sqrt{180} - \sqrt{27} - \sqrt{20} + \sqrt{147} = a(\sqrt{5} + \sqrt{3})$$

$$\sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5}$$

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

$$\sqrt{147} = \sqrt{49 \times 3} = 7\sqrt{3}$$

$$6\sqrt{5} - 3\sqrt{3} - 2\sqrt{5} + 7\sqrt{3} = a(\sqrt{5} + \sqrt{3})$$

$$= 4\sqrt{5} + 4\sqrt{3} = a(\sqrt{5} + \sqrt{3})$$

$$= 4(\sqrt{5} + \sqrt{3}) = a(\sqrt{5} + \sqrt{3})$$

$a = 4$
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3. Without using a calculator and showing all your working, express $\frac{6-\sqrt{8}}{2+\sqrt{8}}$

in the form $a + \sqrt{b}$ where a and b are integers.

$$\frac{6-\sqrt{8}}{2+\sqrt{8}} \times \frac{2-\sqrt{8}}{2-\sqrt{8}}$$

$$= \frac{12 - 6\sqrt{8} - 2\sqrt{8} + 8}{4 - 8}$$

$$= \frac{20 - 8\sqrt{8}}{-4}$$

$$= \frac{8\sqrt{8} - 20}{4}$$

$$= \frac{4\sqrt{2} - 20}{4}$$

$$= \sqrt{2} - 5$$

$$= \sqrt{2} - 5$$

$$= \sqrt{2} - 5$$

$$= -5 + \sqrt{2}$$

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4. Without using your calculator and showing all your working, express

$$\frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} - \sqrt{11}}$$

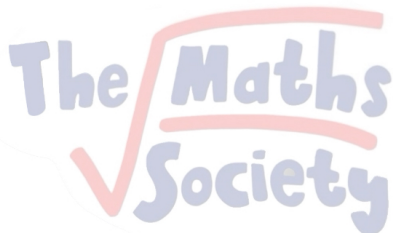
In the form $a + \sqrt{b}$ where a and b are integers.

$$\frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} - \sqrt{11}} \times \frac{\sqrt{13} + \sqrt{11}}{\sqrt{13} + \sqrt{11}}$$

$$= \frac{13 + \sqrt{143} + \sqrt{143} + 11}{13 - 11}$$

$$= \frac{24 + 2\sqrt{143}}{2}$$

$$= 12 + \sqrt{143}$$



5. $\frac{2-\sqrt{2}}{(1+\sqrt{2})^2}$ can be written in the form $a + b\sqrt{2}$ where a and b are integers.

Find the value of a and the value of b .

Show your working clearly.

$$\frac{2-\sqrt{2}}{(1+\sqrt{2})^2} = \frac{2-\sqrt{2}}{(1+2\sqrt{2}+2)}$$

$$= \frac{2-\sqrt{2}}{(3+2\sqrt{2})}$$

$$\frac{2-\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{6 - 4\sqrt{2} - 3\sqrt{2} + 4}{9 - 8}$$

$$= \frac{10 - 7\sqrt{2}}{1}$$

$$= 10 - 7\sqrt{2}$$

$$a = 10$$

$$b = -7$$

6. Without using a calculator and showing all your working, express

$$\frac{4 - \sqrt{12}}{4 + \sqrt{12}}$$

in the form $a - \sqrt{b}$ where a and b are integers.

$$\frac{4 - \sqrt{12}}{4 + \sqrt{12}} \times \frac{4 - \sqrt{12}}{4 - \sqrt{12}}$$

$$= \frac{16 - 4\sqrt{12} - 4\sqrt{12} + 12}{16 - 12}$$

$$= \frac{28 - 8\sqrt{12}}{4}$$

$$= \frac{28 - 16\sqrt{3}}{4}$$

$$= 7 - 4\sqrt{3}$$

$$= 7 - \sqrt{48}$$

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7. Without using a calculator, and showing all your working, express

$$\frac{\sqrt{75} + \sqrt{243}}{\sqrt{7}}$$

in the form \sqrt{a} where a is a positive integer.

Show your working clearly.

$$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$$

$$\sqrt{243} = \sqrt{81 \times 3} = 9\sqrt{3}$$

$$\frac{5\sqrt{3} + 9\sqrt{3}}{\sqrt{7}} \times \frac{-\sqrt{7}}{-\sqrt{7}}$$

$$= \frac{-5\sqrt{21} - 9\sqrt{21}}{7}$$

$$= \frac{-14\sqrt{21}}{7}$$

$$= -2\sqrt{21}$$

$$= \sqrt{21 \times 4}$$

$$= \sqrt{84}$$

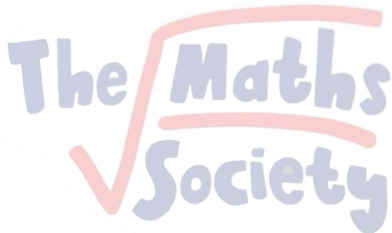
8. Given that m and n are integers, write $\frac{4+\sqrt{20}}{\sqrt{5}-2}$ in the form $m + n\sqrt{5}$
Show each stage of your working clearly.

$$\frac{4+\sqrt{20}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{4\sqrt{5} + 8 + \sqrt{100} + 2\sqrt{20}}{5-4}$$

$$= \frac{4\sqrt{5} + 8 + 10 + 4\sqrt{5}}{1}$$

$$= 18 + 8\sqrt{5}$$



9. Given that $p = \frac{1+\sqrt{5}}{2}$

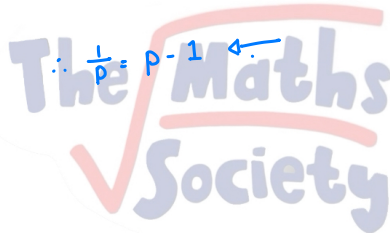
Show that $\frac{1}{p} = p - 1$

Show your working clearly.

$$\frac{1}{p} = \frac{2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}} = \frac{2-2\sqrt{5}}{1-5} = \frac{2-2\sqrt{5}}{-4} = \frac{1-\sqrt{5}}{-2} = \frac{\sqrt{5}-1}{2}$$

$$\begin{aligned} p-1 &= \frac{1+\sqrt{5}}{2} - 1 \\ &= \frac{1+\sqrt{5}-2}{2} \\ &= \frac{\sqrt{5}-1}{2} \end{aligned}$$

$\therefore \frac{1}{p} = p-1$ ←



10. Without using a calculator and showing all your working, express

$$\frac{4-2\sqrt{3}}{\sqrt{3}+1}$$

in the form $a\sqrt{3} + b$ where a and b are integers.

$$\begin{aligned} & \frac{4-2\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{4\sqrt{3} - 4 - 6 + 2\sqrt{3}}{3-1} \\ &= \frac{3\sqrt{3} - 10}{2} \\ &= 3\sqrt{3} - 5 \end{aligned}$$

